

Bayesian magnetotelluric inversion using methylene blue structural priors for imaging shallow conductors in geothermal fields

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ABSTRACT

In geothermal exploration, magnetotelluric (MT) data and inversion models are commonly used to image shallow conductors typically associated with the presence of an electrically conductive clay cap that overlies the main reservoir. However, these inversion models suffer from nonuniqueness and uncertainty, and the inclusion of useful geologic information is still limited. We have developed a Bayesian inversion method that integrates the electrical resistivity distribution from MT surveys with borehole methylene blue (MeB) data, an indicator of conductive clay content. The MeB data were used to inform structural priors for the MT Bayesian inversion that focus on

inferring with uncertainty the shallow conductor boundary in geothermal fields. By incorporating borehole information, our inversion reduced nonuniqueness and then explicitly represented the irreducible uncertainty as estimated depth intervals for the conductor boundary. We used the Markov chain Monte Carlo and a 1D three-layer resistivity model to accelerate the Bayesian inversion of the MT signal beneath each station. Then, inferred conductor boundary distributions were interpolated to construct pseudo-2D/3D models of the uncertain conductor geometry. We compare our approach against deterministic MT inversion software on synthetic and field examples, and our approach has good performance in estimating the depth to the bottom of the conductor, a valuable target in geothermal reservoir exploration.

INTRODUCTION

Magnetotelluric (MT) surveys are commonly used to image crustal resistivity structure at depths ranging from a few hundred meters to several kilometers (e.g., [Gunderson et al., 2000](#); [Ussher et al., 2000](#); [Caldwell et al., 2004](#); [Heise et al., 2007](#); [Bertrand et al., 2015](#); [Wannamaker et al., 2019](#)). From a prospecting perspective, MT surveys in geothermal fields are typically used to resolve the subsurface resistivity structure associated with variations in clay content, temperature, fluid salinity, porosity, and permeability ([Arnason et al., 2000](#); [Cumming, 2009](#); [Bertrand et al., 2013](#); [Trainor-Guitton, 2020](#)). In a geothermal environment, electrically conductive clays are mainly swelling clays (smectite and interstratified illite-smectite), which are collectively referred to as

“smectite” in this paper (e.g., [Cumming, 2009](#)). The presence of smectite in a high-temperature geothermal field can indicate the presence of a shallow, relatively impermeable capping structure, which minimizes mixing of hot reservoir fluids with cooler surface waters. This clay cap is synonymous with the low-resistivity anomaly (LRA) or shallow “conductor” ([Johnston et al., 1992](#)).

The MT method is a passive-source exploration technique that involves measuring time variations in natural electric and magnetic fields at the earth’s surface to determine the resistivity structure at depth. The ratio between the orthogonal components of the electric and magnetic field intensity is known as the characteristic impedance (Z) and represents a measure of the electromagnetic properties of the subsurface. MT inversions seek to estimate the electrical resistivity distribution that best approximates observations of Z .

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Inversion of MT data has primarily relied on gradient-based deterministic methods, and these have been deployed across a wide range of settings (e.g., Hautot et al., 2000; Heise et al., 2008; Constable et al., 2009). However, the resistivity structure output by these methods can depend on the starting model, suggesting that a local rather than a global solution has been achieved (e.g., De Groot-Hedlin and Constable, 1990). In typical deterministic inversions, the resistivity structure is regularized. For example, in the Occam method (Constable et al., 1987), a maximally smooth model is found within the data noise. Geophysical inverse problems that rely on surface observations are nonunique; thus, an infinite number of models can explain the observed data. Regularization is introduced to eliminate this nonuniqueness so that the inversion algorithm can converge to a solution. However, nonuniqueness and the implied uncertainty can be useful to decision makers where it genuinely reflects a deficit of knowledge. Previous approaches to quantifying uncertainty in deterministic inversions relied on the calculation of the covariance matrix of unknown parameters distilled from the model Jacobian derived at the optimal solution using the covariance of the observations (Auken and Christiansen, 2004). However, this approach can be too simple for complex inverse problems and could underestimate the true uncertainty (Tarantola, 2005; Trainor-Guitton and Hoversten, 2012).

Stochastic inversions for geophysical methods provide a complementary alternative to deterministic approaches (e.g., Tarantola, 2005; Chen et al., 2008). Computational requirements are usually the primary limitation, preventing their application to problems with large numbers of parameters. Nevertheless, they can provide more detailed uncertainty information about the estimated parameters and the solutions tend to be less biased by the initial conditions (Chen et al., 2008; Trainor-Guitton and Hoversten, 2012). One approach to uncertainty quantification in stochastic inversion relies on the calculation of marginal posterior distributions from several independent samples (models). The samples are generated using Markov chain Monte Carlo (MCMC) methods based on Metropolis-Hastings algorithms (Metropolis et al., 1953; Hastings, 1970), in which probabilities for each sample are estimated by a Bayesian formulation. Bayesian sampling-based inverse methods estimate the model parameter uncertainty by generating an ensemble of plausible models from which distributions for estimated parameters can be inferred. Regardless of the complexity of the inversion, the examination of the posterior probability distribution is generally a valid approximation of the true uncertainty (Tarantola, 2005).

Given the computational cost of MT Bayesian inversion, 1D implementations have been more common (Guo et al., 2014; Mandolesi et al., 2018; Xiang et al., 2018; Blatter et al., 2019). The variations usually differ on how sampling for constructing the Markov chains is performed. The transdimensional (trans-D) sampling technique has proven useful when the number of layers that define the model parameters is unknown (Mandolesi et al., 2018; Blatter et al., 2019) and for error correlation analysis (Guo et al., 2014; Xiang et al., 2018). The Hamiltonian sampling algorithm has also been implemented for estimating sharp and smooth transitions between formations by the use of prior distributions (Conway et al., 2018). Bayesian 2D and 3D inversions have proven effective for sharp boundary parameterization. Sharp boundary parameterization seems to be useful when resistivity structures present sharp contrasts in the vertical direction (Auken and Christiansen, 2004) as they tend to in geothermal fields (Kumar et al., 2010; Chen et al., 2012; Rosas-Carbajal et al., 2015).

In geothermal exploration, resistivity models derived from deterministic MT inversions have played a leading role in defining drilling targets during the early stages of a project (Sepulveda et al., 2012). It is particularly useful that they constrain the depth and extent of a shallow conductor presumed to be a clay cap (Johnston et al., 1992). Whereas 3D MT inversions are useful in studying field-wide variations of resistivity, 1D MT inversions are commonly used for a more detailed examination of the shallow conductive layers (Cumming and Mackie, 2009). The application of MT relies significantly on conventional conceptual models of resistivity (Arnason et al., 2000; Cumming, 2009) where upflow zones in the reservoir are traditionally associated with a doming shape of the bottom of the conductor. At the same time, MT inversions help to improve conceptual models and our understanding of the hydrogeologic implications of the resistivity structure. Finally, MT inversions can be integrated with complementary geophysical data such as magnetics (Sepulveda et al., 2012), drill-hole data sets such as methylene blue (MeB) or lithostratigraphy (Gunderson et al., 2000; Correa et al., 2013), and temperature logs (Sewell et al., 2012) for a richer picture of the subsurface.

MeB titration is an analysis technique used in geothermal exploration to estimate the smectite content along the borehole path in drilling returns (Gunderson et al., 2000; Rosenberg, 2017). Smectite-rich formations in geothermal fields usually present a very low electrical resistivity (approximately 1–5 Ωm) due to the high cation exchange capacity of the smectite (Arnason et al., 2000). The systematic presence of smectite detected by MeB in well cuttings is expected to be correlated with the shallow conductor imaged by MT surveys of a geothermal field (Gunderson et al., 2000). Although it does not constrain absolute resistivity, a MeB profile is nevertheless an indirect indicator of contrasts in electrical resistivity.

In this study, we develop a 1D MT Bayesian inversion that incorporates MeB data as a constraint. We use lithologic information of the conductive clay content in wells given by MeB data to construct structural prior distributions for the 1D MT Bayesian inversion. The inversion is formulated in the context of geothermal fields to infer under uncertainty the shallow conductor boundary using a three-layer model. The structural priors are defined for the second layer boundary, a zone that represents the conductive clay cap under each station. Then, 1D models are interpolated to construct pseudo-2D sections. We test the inversion on a synthetic example and then using observed data from the Wairakei geothermal field in New Zealand. For comparison, we also invert the synthetic and field data using a widely used deterministic MT inversion method (ModEM, Kelbert et al., 2014), obtaining consistent results.

Our goal in developing this method is to provide a means of propagating uncertainty through the inverse problem so that the output is useful to decision makers. Furthermore, by incorporating information obtained by direct observations in wells, we strengthen the connection between the model and reality while reducing uncertainty. The methods could improve conceptual models of shallow geothermal hydrology, provide additional constraints for geothermal reservoir simulations, and support decision-making processes to define drilling targets or field management strategies.

The Wairakei geothermal field

The Wairakei geothermal field is located in the Taupō Volcanic Zone (TVZ) in the North Island of New Zealand (Figure 1). Situated at the southern extension of the Tonga-Kermadec subduction system,

the TVZ is a rifted continental arc of approximately 300×60 km in dimension (Wilson et al., 1995). The rift's extension rate is approximately 7–16 mm/year and is oriented in the northwest–southeast direction (Wallace et al., 2004). The TVZ formed in response to the westerly oblique subduction of the Pacific Plate beneath the Australian Plate along the Hikurangi Margin (Cole and Lewis, 1981). Comprehensive reviews of the geologic structure of TVZ can be found in Cole et al. (1990), Rowland et al. (2010), and Seebeck et al. (2014).

The Wairākei geothermal field is located to the northeast of Lake Taupō in central TVZ, and it is delineated by a resistivity boundary derived from a direct current (DC) electrical resistivity survey (Figure 1; Risk, 1984). Wairākei presents a complex lateral and vertical variation in lithologic extent and thickness. The stratigraphy has been reviewed by Rosenberg et al. (2009) and updated by Bignall et al. (2010).

Major electrical resistivity studies in Wairākei by geophysical methods include DC (Risk, 1983-1984; Bibby et al., 1995), long- offset transient electromagnetic (Caldwell et al., 1999), and MT (Sepulveda et al., 2012). Wairākei has been a proving ground for electrical resistivity methods for geothermal exploration. It was here that large, sharp contrasts in shallow resistivity demarcated conductive geothermal ground from a more resistive surrounding (Bibby et al., 2009) leading to the concept of a resistivity boundary that outlined a geothermal field. Recently, spatial correlations between the presence of clays in well returns (from MeB data) and a shallow conductor revealed by MT surveys have been reported (Sepulveda et al., 2012; Correa et al., 2013). Comparisons with temperature logs have distinguished cold (or relict) and hot clays, with the former thought to correspond to past alterations when temperatures were higher (Sepulveda et al., 2012). Reviews of other geophysical methods applied at Wairākei can be found in Hunt et al. (2009) and Bromley et al. (2018).

METHODOLOGY

The following section details our MT inversion formulation (Figure 2). The key innovation is the use of clay content distribution as a priori information derived from MeB data in the MT inversion. MeB data contain direct information of the vertical extent of the clay cap at the location where wells are drilled. First, we model the vertical extent of the clay cap and estimate the depths of the clay cap boundaries from the MeB data. Then, an interpolation is performed to estimate the depths beneath MT station locations for use as a structural prior during inversion (Calvetti and Somersalo, 2018). A Bayesian inversion is then undertaken to quantify uncertainty in the model parameters with the structural prior providing additional discrimination of the plausible models.

The approach is pseudo-2D/3D in the sense that 1D conductivity profiles are constructed beneath each MT station, with interpolation used to infer structure in between. For the MT inversion beneath each station, we used a simplified three-layer model of the shallow subsurface that

explicitly prioritizes imaging the first-order conductivity feature, presumed to be a shallow conductive body that corresponds to the clay cap. We used the Bayesian inversion to obtain a posterior probability distribution over the five parameters of this model. From a Bayesian perspective, we are constructing prior beliefs of the clay cap depth boundary from MeB data. We then update them by assimilating the MT data through inversion, to finally infer the most probable depth boundary for the conductor (that is in turn reflective of the clay cap).

Structural priors for layer depth from MeB data

To provide an initial estimate of the clay cap depth, we modeled MeB data profiles in wells to estimate probable distributions for boundary locations. Because we wish to capture the sharp layer structure, we used a square function as the model (e.g., Figure 3, the model parameters are z_1 , depth to the top of the clay cap; z_2 , depth to the bottom of the clay cap; and c , smectite clay content indicate by MeB data) with the edges of the square representing the appearance and then disappearance of clay in the vertical profile. We used the MCMC method (Foreman-Mackey et al., 2013) and a uninformative prior to estimate a posterior distribution over z_1 , z_2 , and c as

$$P(z_1, z_2, c) \propto \exp\left(-\frac{1}{2} \left\| \frac{\mathbf{c}_{\text{obs}} - \mathbf{c}_{\text{est}}}{\mathbf{c}_{\text{error}}} \right\|_2^2\right), \quad (1)$$

where $\mathbf{c}_{\text{obs}}$ is the MeB observations and $\mathbf{c}_{\text{est}}$ is the output of the square model, which depends on choices of z_1 , z_2 , and $\mathbf{c}$. In the absence of multiple tests or error information supplied with the data,

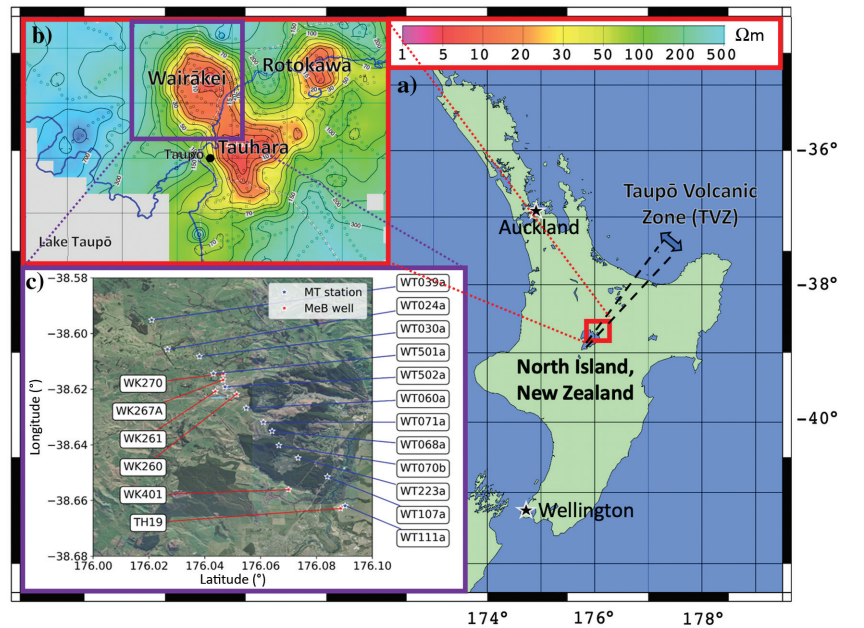


Figure 1. (a) Map of the North Island of New Zealand, with the location of the Taupō Volcanic Zone. (b) The red square shows the Wairākei geothermal field location and a surface resistivity map (approximately 250 m depth) derived from previous DC studies (Risk, 1984). In the resistivity map, the red to orange area delineates the traditional boundary of the Wairākei geothermal field. (c) The purple square shows the location of the data set containing MT stations (the blue stars) and wells with MeB data (the red stars) above the topography.

we used an error c_{error} of 2%, which is the minimum resolution of the titration procedure (One drawback of using MeB data is the absence of measurement error that could improve the estimation by giving more weight to specific data points. If repeat measurements of MeB on well cuttings were performed, a simple estimation of data error for the analytical technique could be achieved, keeping the error in cutting depth). The posterior $P(z_1, z_2, c)$ describes an ensemble of models that are consistent with the data from which marginal distributions for individual parameters can be constructed (Figure 3). For example, the marginal distribution $P(z_1)$ provides an uncertain estimate of the depth to the top of the clay cap.

Wells in geothermal fields are not generally in the same location as MT stations (e.g., Figures 1 and 2). Therefore, interpolation is necessary to estimate priors at station locations (Figure 4a). We do this by first parameterizing marginal distributions for z_1 and z_2 at well (i) as normal distributions with the mean $\mu^{(i)}$ and standard deviation $\sigma^{(i)}$. Then, for an MT station (k) between adjacent MeB wells (i) and ($i+1$), the most likely position of the layer boundary $\mu^{(k)}$ is estimated as the distance-weighted average of $\mu^{(i)}$ and $\mu^{(i+1)}$ (Figure 4b). Similarly, the standard deviation $\sigma^{(k)}$ is calculated as a weighted average of distance-penalized standard deviations $\sigma^{(i)}$ and $\sigma^{(i+1)}$ (Figure 4c). The penalization models how uncertainty (σ) increases with distance from a well. We have used a function that linearly increases σ with distance so that

$$\sigma^{(k)} = \sum_i^{N_{\text{wells}}} w^{(k,i)} (\sigma^{(i)} + \lambda d^{(k,i)}), \quad (2)$$

where N_{wells} is the number of wells used to construct the prior at the station position, $d^{(k,i)}$ is the distance between the position of the well i and the position of the station k , and λ is a correlation length that models the increase in uncertainty (in meters). We tried different values for λ and obtained reasonable inversion results between 20 and 80 m/km. The weight function is calculated as $w^{(k,i)} = d^{(k,i)} / D$, where $D = \sum_i^{N_{\text{wells}}} d^{(k,i)}$ is the sum of the distances from the station to the wells considered.

An alternative method for interpolating uncertainty could be using kriging (Chilès and Desassis, 2018). Kriging guarantees that interpolated MeB priors match exactly with the inversion at the locations of wells. In our approach (equation 2), the MeB prior will not necessarily match exactly, although the relative sparsity of MeB wells means that they will be the dominant influence. In isolated cases, a cluster of wells (typical in geothermal fields) will present conflicting MeB profiles. In these cases, our averaging approach avoids large spatial changes in the interpolation over small distances while still guaranteeing a spatially smooth prior. However, we acknowledge that other methods could provide an improved interpolation.

Finally, the structural prior for a layer boundary depth at an MT station (k) is

$$p(z_j) \propto \exp\left(-\frac{1}{2} \left\| \frac{z_j - \mu_j^{(k)}}{\sigma_j^{(k)}} \right\|_2^2\right), \quad (3)$$

where $j = 1, 2$ corresponds to the layer boundary and z_j is the depth to the layer boundary.

Three-layer resistivity model for MT inversion

For the inversion at each MT station, the parameters of a three-layer resistivity model are determined: the thickness and resistivity of the top two layers (h_1, ρ_1 and h_2, ρ_2) and the resistivity of the lower half-space (ρ_3). The second layer explicitly represents the shallow conductor.

The layer model allows for arbitrarily sharp resistivity boundaries, compared with regularized inversion methods that solve for the smoothest model away from a priori resistivity distribution. Thus, it is suitable for capturing the abrupt appearance of clay in a vertical profile. In reality, we could expect some degree of smoothing at the boundaries where the clay alteration is ongoing. For calculating synthetic observations from the three-layer model, we used the 1D layered-earth formulation (Cagniard, 1953; Weidelt, 1972; Pedersen and Hermance, 1986; Basokur, 1994; Niwas et al., 2005) to estimate the impedance surface response.

The three-layer model can be expressed as $\hat{\mathbf{p}} = F(\mathbf{p}, \mathbf{z})$ with $\hat{\mathbf{p}}$ corresponding to the MT data across all frequencies, $\mathbf{z}$ being the unknown

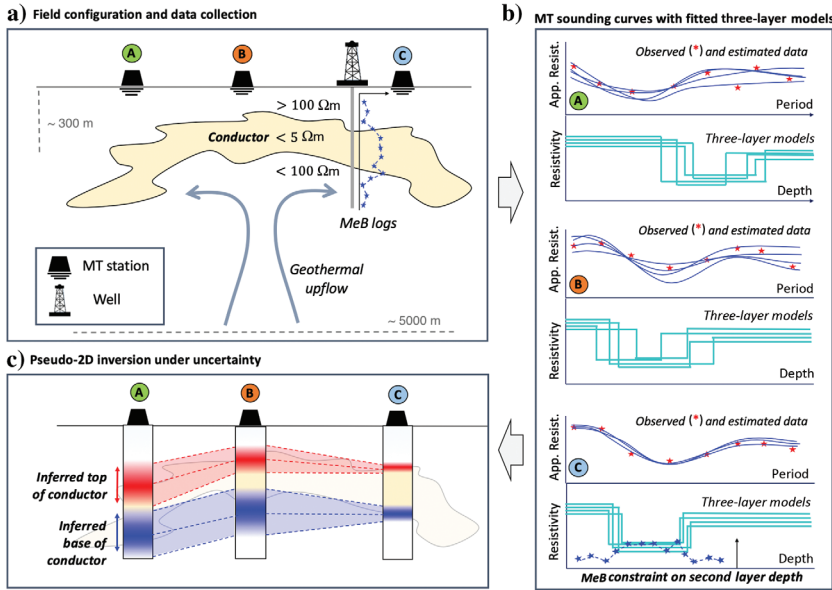


Figure 2. Schematic outlining the Bayesian inversion method. (a) A conceptual model of an MT survey deployed over a geothermal field, including stations A, B, and C. The presence of the LRA or shallow conductor (the yellow shading) is highlighted where an MeB log from a drilled well is drawn. The MeB profile (the blue stars) reveals the presence of smectite. (b) MT signals (the red stars) are inverted stochastically using a 1D three-layer resistivity model in which the second layer is constrained to have low resistivity. From the stochastic inversion, multiple models (the light blue profiles) consistent with the data (the dark-blue profiles) are obtained. MeB logs are used to constrain the location of the interfaces, introducing lithologic information into the inversion. If the MT station is located close to a well, such as station “C,” the influence of the MeB logs is stronger and there is less uncertainty on the estimated conductor boundaries. (c) After the inversion, uncertain envelopes for the top and bottom boundaries of the conductor are constructed by interpolating between the stations. Under each inverted station, depth distributions over the boundary parameters are drawn. A reduction in uncertainty is emphasized in station C as a result of the use of MeB logs from the well as a constraint.

depths of layer boundaries, ρ the unknown layer resistivity values, and $F(\cdot)$ the forward model. This model explicitly acknowledges the dominant conductivity feature while otherwise keeping the number of parameters to a minimum, a valuable property when analyzing multidimensional posteriors. Because the inversion seeks a three-layer resistivity model, estimation of the second-layer thickness is always performed. When this second layer is not necessary to fit the data, the inversion reduces its thickness until it has a negligible effect on the result. An alternative method for discriminating between the preference for three- or two-layer model could be implementation of reversible-jump or trans-D McMC algorithms (Ray et al., 2013; Blatter et al., 2018; Mandolesi et al., 2018).

Bayesian formulation of the MT inversion

The MT data at each station are inverted to determine the set of three-layer models that plausibly fit the log magnitude and phase of the off-diagonal impedance tensor terms Z_{xy} and Z_{yx} . We used an McMC algorithm to explore the 5D parameter space and quantify an approximate posterior distribution. We used the McMC ensemble sampler implementation presented in Foreman-Mackey et al. (2013), which is based on the affine-invariant algorithm given by Goodman and Weare (2010).

We follow the Bayesian approach for estimating the joint posterior distribution $P(\cdot)$ over the model parameters $[\rho, \mathbf{z}]$

$$P(\rho, \mathbf{z} | \hat{\rho}) \propto L(\hat{\rho} | \rho, \mathbf{z}) \prod_{i=1}^{N_{\text{layers}}} p(\rho_i) \prod_{j=1}^{N_{\text{layers}}-1} p(z_j), \quad (4)$$

where the first factor on the right side is the likelihood function $L(\cdot)$ that relates the MT observations $\hat{\rho}$ to a particular resistivity model

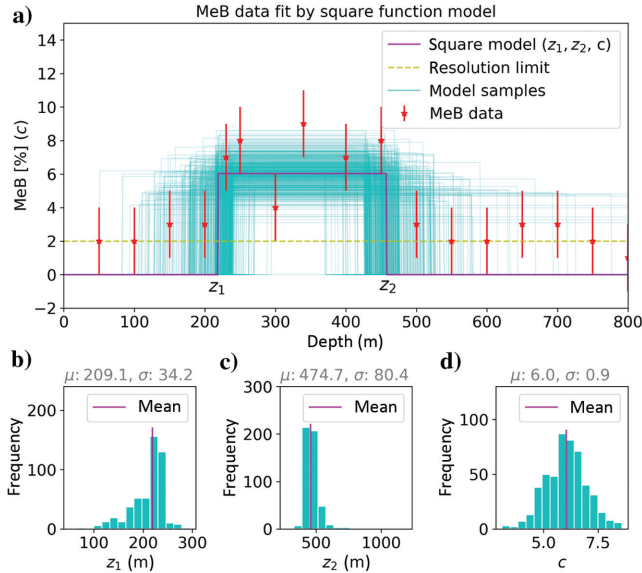


Figure 3. Methylene blue (MeB) inversion results for geothermal well WK261 (Figure 1) obtained by fitting a square function to the MeB data (a). The purple profile indicates the best-fitting model to the data (red error bars) whereas cyan profiles are samples from the posterior. Each model is defined by z_1 , z_2 , and c parameters. Lower panels show marginal posterior distributions for square model parameters z_1 (a), z_2 (c) and c (d).

$[\rho, \mathbf{z}]$. The remaining factors $p(\cdot)$ are the prior distributions for the layer resistivity values $p(\rho_i)$ and the structural priors derived from MeB data for thickness parameters $p(z_j)$ (equation 3).

Prior distributions for layer resistivity

Conceptual models of resistivity for geothermal reservoirs often include the shallow conductor on top of the main reservoir that is assumed to be associated with a low-permeability clay cap (e.g., Johnston et al., 1992). The dominant clay in these formations is usually smectite, which presents with a resistivity lower than 10 Ωm (Anderson et al., 2000) and commonly between 1 and 5 Ωm (Arnason et al., 2000; Gunderson et al., 2000). We introduce this information as a uniform prior distribution that constrains the range of possible resistivity values assigned to the middle layer. For the upper and lower layers, the resistivity is allowed to vary between 5 and 10⁵ Ωm . The uniform prior distribution for the resistivity of the second layer is given by

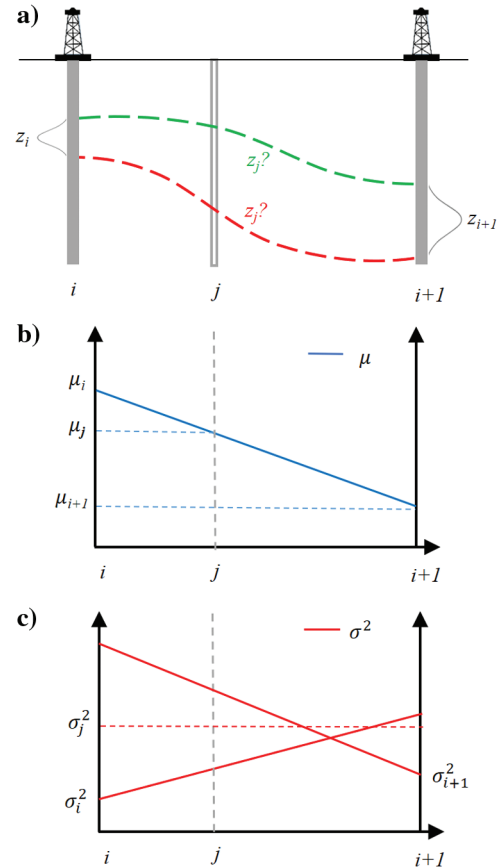


Figure 4. Schematic of uncertainty interpolation for location j between the estimated depth intervals at locations i and $i+1$. The distribution estimated at “ j ” is used as a prior distribution for the MT inversion. (a) The colored dashed lines show the possible interface positions given inferred distributions at “ i ” and “ $i+1$.” (b) Interpolation of μ (mean) of the boundary distribution at j between inferred μ at i and $i+1$ as a weighted average between bounding values. (c) Interpolation of σ^2 (variance) at j between inferred values at i and $i+1$. The calculation applies a linear distance-weighted penalty and takes an average between the bounding values. The intent is to model the increase of uncertainty of the boundary depth with the greater distance from the wells.

$$p(\rho_2) = U(1, 5), \quad (5)$$

where U denotes a uniform distribution. We recommend that the edges of the uniform prior be determined on a case-by-case basis for the field under study. For the Wairakei geothermal field, these edges are assigned the values in equation 5 where the clay cap formation has been associated with a resistivity value lower than $5 \Omega\text{m}$ (Sepulveda et al., 2012).

Likelihood and posterior probability distribution

The likelihood function quantifies the misfit between the MT data and the estimated observations generated by a forward model. We express the log of the likelihood function as a weighted sum of square misfits between the data observed and estimated at each MT station

$$L \propto \exp\left(-\frac{1}{2} \sum_{t=1}^{N_{\text{periods}}} \left\| \frac{\hat{\rho}_t^{\text{obs}} - \hat{\rho}_t^{\text{est}}}{\sigma_t^{\text{obs}}} \right\|_2^2\right), \quad (6)$$

where $\hat{\rho}_t^{\text{obs}}$ and $\hat{\rho}_t^{\text{est}}$ are the observed and simulated data for the period t and σ_t^{obs} is the measurement error. The inverted data correspond to the $\log_{10}(\cdot)$ of the impedance magnitude $|Z|$ and phase ϕ (following Wheelock et al., 2015) for the off-diagonal impedance tensor terms Z_{xy} and Z_{yx} .

Figure 5 shows models inverted from one MT station (WT111a in Figure 1) with and without imposition of the MeB structural prior. For both inversions, the fit to the data by the three-layer model is very good at periods shorter than 2 s with an average root-mean-square (rms) (chi-square misfit; Pearson, 1900) of approximately 1.5 (Figure 5a and 5b). However, inspection of the posterior marginal distributions shows significant differences for some para-

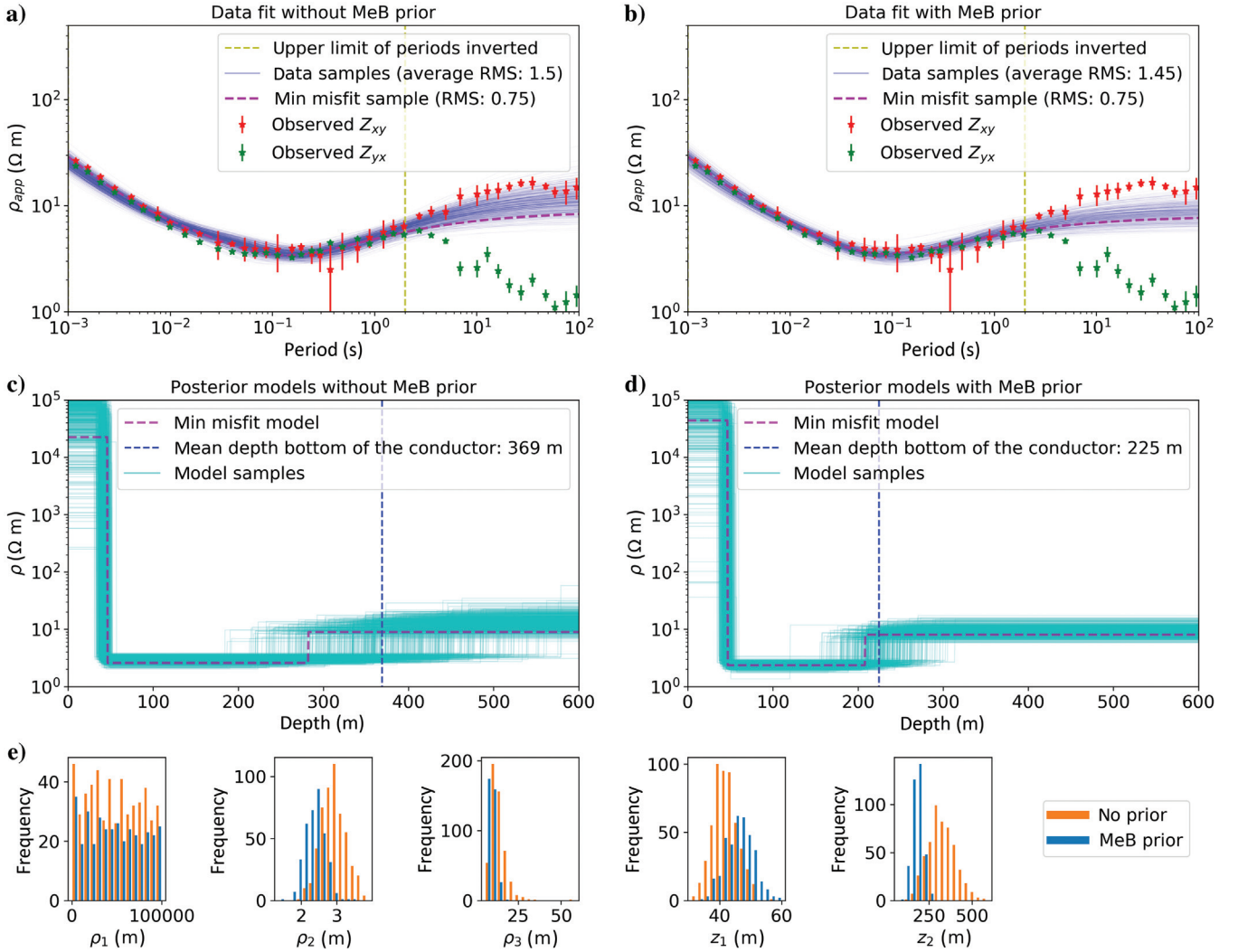


Figure 5. Comparing data fit and marginal posterior distributions from the stochastic 1D inversion at station WT111a (Figure 1) (b and d) with and (a and c) without structural priors. The rms corresponding to a chi-square misfit is also given (Pearson, 1900). (a and b) Nondiagonal components of the impedance tensor (the red and green stars) along with the best-fitting model (purple) and posterior samples (blue). Models are fit using data for periods less than 2 s (the dashed yellow line). (c and d) Sample models from the posterior for inversions in (a and b) with the mean depth of the lower LRA boundary indicated (the dashed blue line) and minimum misfit sample (the dashed magenta line). (e) Marginal posterior distributions for inverted parameters with (blue) and without (orange) the MeB structural prior. The three-layer model parameters: z_1 (the first layer thickness), z_2 (the second layer thickness), ρ_1 (the first layer resistivity), ρ_2 (the second layer resistivity), and ρ_3 (the third layer resistivity).

meters (Figure 5c–5e). Using a MeB prior biases the middle layer to narrower thicknesses and considerably reduces uncertainty (Figure 5e). Biases in the z_1 and ρ_2 parameters are also introduced. Thus, incorporating prior information and inspecting marginal distributions individually can be more informative than bulk measures such as the rms misfit.

Posterior distributions for z_2 generally present a greater variability than for z_1 (Figure 5e), which reflects an increased uncertainty associated with depth parameters below the first model layer. This is expected because the thickness of the uppermost resistive layer is easy to resolve whereas subsequent layer resolution is dependent on the conductance — a product of conductivity and layer thickness — of overlying layers and obscured by the attenuating effects of intermediate conductors. Nonuniqueness arising through trade-offs between absolute conductivity and layer thickness is somewhat managed through priors on the clay conductivity and bottom boundary depth estimation using MeB.

Inverting for a range of periods, convergence, and modeling observational error

To be consistent with the 1D three-layer model, MT data at each station are inverted for a specified range of periods. The set of MT data from Wairakei is primarily 1D from its lowest periods to 1–10 s (depending on the station). For example, station WT111a (Figure 5) is inverted up to 2 s, above which 2D or 3D behavior is observable as a split between Z_{xy} and Z_{yx} (in ρ^{app} and ϕ). However, at periods of less than 3 s, roughly equivalent to a skin depth of 3 km (assuming average shallow resistivity of 10 Ω m), the 1D approximation is

valid. This analysis needs to be performed for every station, and a range of periods should be selected for each inversion accordingly.

For constructing the chains at each MT inversion (for each station independently), we considered 36 walkers and 3500 jumps. After discarding the “burn-in” section (approximately 500 jumps for the inversion shown in Figure 5), the autocorrelation time is estimated, which is the number of jumps in the chains required to generate an independent sample of the target density (approximately 400 jumps for the inversion shown in Figure 5). To ensure convergence, we ran three parallel chains (starting from different initial models) and we compared the resulting marginal posterior distributions. We also compared marginal posteriors for the first three sections of the chain. In both cases, we obtained indistinguishable distributions.

Error in the likelihood function σ_t^{obs} is modeled as a mixture of observation and model error:

$$\sigma_t^{obs} = \max[\sigma_t^{obs}, \gamma \hat{\rho}_t^{obs}], \quad (7)$$

where γ is set to 10% and 5% for $\log_{10}(|Z|)$ and ϕ , respectively (t for the period). This is commonly known as the error floor. Equation 7 expresses the prioritization of observation error σ_t^{obs} , except in situations in which this is very small. Then, a minimum model error term is retained ($\gamma \hat{\rho}_t^{obs}$), which reflects an anticipated irreducible misfit with the data due to the use of a simplified model. The value γ is uncertain and likely varies depending on the simplicity of the model. Processed MT data can contain unrealistically small errors (e.g., <5% of its magnitude) at data points at which a strong noise source compared to the natural signal level is present across a par-

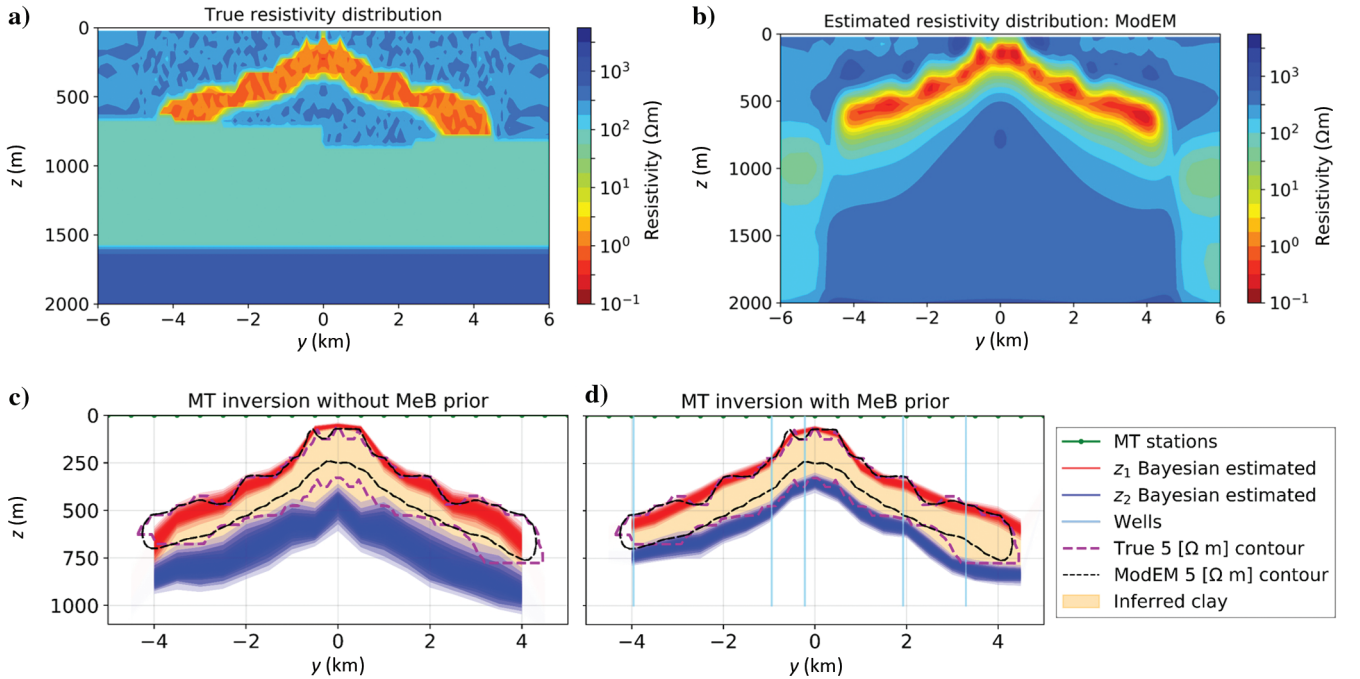


Figure 6. MT inversion in a synthetic geothermal field comparing the results from our MT Bayesian inversion with/without structural priors derived from MeB data and the deterministic MT inversion software ModEM (Kelbert et al., 2014). (a) The true synthetic resistivity distribution characterized by a shallow conductor. (b) Inferred resistivity distribution from the 2D deterministic inversion using ModEM. (c and d) Results for our MT Bayesian inversion with/without MeB structural priors. Uncertain envelope show the inferred top and bottom boundary distributions for the shallow conductor. The envelopes cover the probability of occurrence of 5%–95% for the interfaces. What lies between the two boundaries is inferred to be the clay cap. Comparison with the (a) true model and (b) ModEM inversion model is also shown, where the black and magenta lines show the 5 Ω m contour typically associated with the clay cap boundary.

ticular period band. These small errors can cause an overfit in equation 6 for these “non-MT” data points at which the assumption of a uniform plane-wave source is violated and could result in an artificially rough model (Parker 1980). It has also been shown that these unrealistic errors can occur due to uncertainties in the MT data that are not only derived from the time series processing but also from geologic noise (Li et al., 2003).

SYNTHETIC EXAMPLE

To test the accuracy of our inversion, we apply it to a 2D synthetic resistivity distribution that approximates a geothermal system. The

results are compared to a deterministic inversion obtained using ModEM (Kelbert et al., 2014).

True resistivity model, MT data, and MeB priors

We generate a 2D synthetic resistivity distribution with a shallow doming shape conductor that is reflective of a real geothermal field (Figure 6a). The resistivity distribution was generated from the mapping of a synthetic geothermal temperature distribution (Dempsey et al., 2016; Ardid et al., 2018). The model resistivity and depths are assumed to be representative of MT inversions in NZ geothermal fields (Heise et al., 2008; Sepulveda et al., 2012; Sewell et al., 2012; Bertrand et al., 2015).

Using this reference distribution (Figure 6a), MT surface responses at the station locations were computed by forward modeling using ModEM in forward mode. The 2D impedance Z is assumed to be measured at discrete periods between 0.001 and 500 s. MT stations were positioned with spacing of 500 m out to 10 km from the center of the grid ($y = 0$ km). Once the complex impedance was calculated, 5% normally distributed random noise was added.

To test the advantage of including geologic information in the stochastic MT Bayesian inversion, we also constructed synthetic MeB priors. First, we generated clay cap boundary distributions at five wells located randomly, emulating the results from MeB data inversions (equivalent to the clay cap boundary distributions shown in Figure 3b and 3c). The prior distributions were constructed as normal distributions centered at the depth of the 5 Ω m resistivity intersection and with a standard deviation of 60 m (similar to observations from the Wairakei MeB data; Figure 3). Then, boundary distributions at MT station locations are interpolated using the method described earlier.

MT inversion results

Figure 6b shows the inferred resistivity distribution obtained by deterministic inversion with ModEM. Figure 6c and 6d shows the inferred uncertain conductor boundary derived from our inversion without the use of structural priors (Figure 6c) and then again including these (Figure 6d). To visualize uncertainty for each boundary, we show an envelope that encloses 5%–95% of the posterior models with the clay cap assumed to lie between these two boundaries.

Comparison between the stochastic and deterministic inversions

The stochastic and deterministic methods largely recover the geometry of the shallow conductor. The deterministic inversion (Figure 6a) recovers a diffusive version of the conductor, which is due to the regularization scheme used

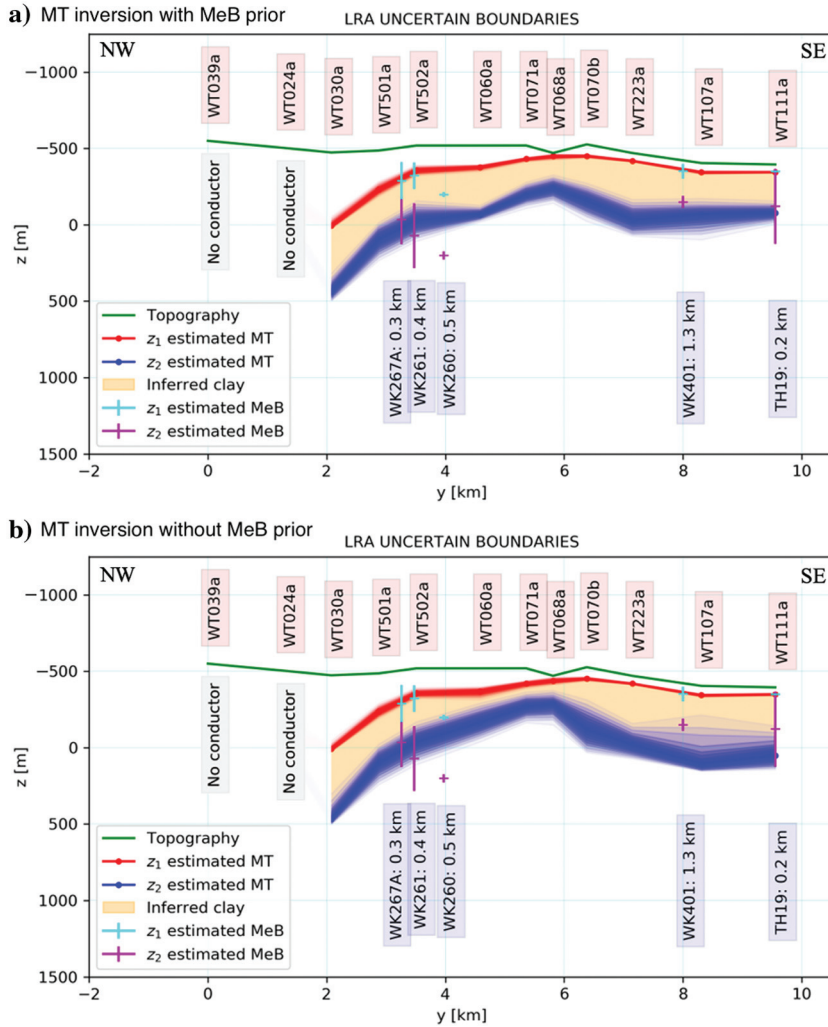


Figure 7. Uncertain boundary distributions of the inferred shallow conductor (the top and bottom boundary) for the Wairakei MT profile (Figure 1). (a) Inversion result incorporating MeB structural priors ($\lambda = 40$ m; equation 2). (b) Inversion result without MeB structural priors. The shaded envelopes for the top (red) and bottom (blue) boundaries are plotted covering between 5%–95% of occurrence probability for the interface. Interfaces correspond to the z_1 and z_2 parameters in the three-layer resistivity model. What lies inside both boundaries is inferred as the clay cap. The distribution of clay cap boundaries estimated from MeB data inversion (from the wells shown in Figure 1) is plotted for the top (cyan) and bottom boundaries (magenta). The red boxes at the top indicate the name of the MT stations inverted. The blue boxes at the bottom indicate the names of the MeB wells along with the perpendicular distance to the profile. The green line shows the topography.

to converge the inversion. On the other hand, the stochastic inversion (Figure 6c and 6d), because it focuses on finding the boundaries of the conductor, delivers uncertain interfaces.

To compare the accuracy of deterministic and stochastic inversions, 5 Ωm resistivity contours are extracted from the reference distribution (Figure 6a) and from the deterministic model (Figure 6b). This contour was selected because it is often used as an indicator for a dominant smectite presence (Arnason et al., 2000; Gunderson et al., 2000).

Both inversions recover the depth of the top boundary reasonably well, with the reference 5 Ωm contour closely followed by the deterministic model and the uncertain envelope of the stochastic inversion mostly bounding it (Figure 6c and 6d).

The bottom of the conductor is generally more difficult to locate accurately from surface MT observations. The deterministic inversion does a better job of recovering the bottom boundary for sections of the conductor approximately -2 and 2 km but underestimates its depth near the center (approximately 65 m shallower than the true value; Figure 6c and 6d). In other areas, the stochastic inversion without structural priors tends to overestimate its depth (although it captures it within the uncertainty at many locations; Figure 6c). When MeB priors are used, the inference of the base of the conductor is substantially improved (Figure 6d). This illustrates in a simple fashion how geologic information can be used to improve estimation of resistivity boundaries.

FIELD EXAMPLE

In this section, we describe an application of our inversion to the Wairakei geothermal field. We use a set of data comprising an MT survey and MeB data in wells from the geothermal area. For a north-west–southeast profile covering 12 MT stations and six wells with MeB data (Figure 1), the uncertain top and bottom boundaries of the shallow conductor are estimated (Figure 7).

The inversion recovers a consistent thickness for the clay cap, averaging approximately 300 m (Figure 7). The northwest section of the profile beneath stations WT039a and WT024a is interpreted as an absence of a clay cap. This absence is consistent with Figure 1, where the stations (WT039a and WT024a) are identified to be outside the main reservoir area, as delineated by a previous DC resistivity survey. To the east, a thicker conductor appears (beneath station WT030a) consistent with previous inversions for the field (Sepulveda et al., 2012) and with conceptual models of resistivity, where upflow zones are associated with a doming shape (Figure 2).

Inversion results with and without use of the structural prior are shown in Figure 7a and 7b, respectively. Both inversions fit the data equivalently (average rms values of 1.64 and 1.65, respectively), but the boundary envelopes present significant differences. East of station WT502a, a regular thickness and less uncertainty are obtained when priors are used. Whereas the top boundary is recovered at similar depths, the bottom of the conductor varies significantly between inversions, supporting the idea that incorporation of the structural prior is particularly sensible for this interface.

The average uncertainty associated with boundary depths is quantified for a range of λ values (equation 2) in Table 1. For the top boundary, similar values are obtained as λ decreases suggesting that the use of priors does not decrease the uncertainty meaningfully when estimating the top boundary. On the other hand, the average standard deviation at the bottom boundary reduces con-

siderably as the λ parameter is decreased, showing that MeB priors are reducing uncertainty.

Comparison between stochastic and deterministic inversions

A deterministic 3D inversion was also performed using the ModEM software (Kelbert et al., 2014) for the same Wairakei MT data set (Figure 8). The data (north–south orientation) were inverted for the full impedance tensor and for the full range of periods (approximately between 0.001 and 1000 s). The model dimensions for the first layer were $289 \times 289 \times 30$ m; then, the thicknesses of the layers increase with depth. An error floor of 5% of Z_{xy} and Z_{yx} and 10% of Z_{xx} and Z_{yy} was applied, resulting in a final rms misfit of 2.4.

A clear, but discontinuous, conductor is observed in the shallow section (the first 500 m) deepening beneath station WT030a and absent below WT024a and WT039a (Figure 8a). Lateral discontinuities in the shallow conductor are potentially artifacts related to the distance between stations (up to 1 km) and the inversion regularization (smoothing) applied. We note that our method is unable to resolve discontinuities, real or otherwise, due to its reliance on interpolation. However, overall, we assess that there is a reasonable degree of similarity between both inversion models with regard to the main features of the shallow conductor.

Both inversion models tend to agree with the absolute resistivities of the conductor (e.g., Figure 7b–7e). However, in some locations, there is up to one order of magnitude difference in the inferred resistivities in the shallow sections (up to 100–150 m depth) possibly related to the lack of resolution on the first layer resistivity of our inversion (as shown in Figure 5a). Clear boundaries in the deterministic inversion are sometimes difficult to observe, whereas these are more obvious in the stochastic inversion. Of course, this is to be expected when the stochastic inversion enforces the use of a coarse block model and the deterministic inversion uses a dense grid with vertical regularization that prefers smooth gradients.

The top boundary of the inferred conductor is well recovered by both inversions. However, for the bottom of the conductor, the deterministic inversion shows a smooth resistivity transition that makes it difficult to pick a clear boundary. Nevertheless, there is a correlation between both inversions, with the depth extent of the bottom boundary from the stochastic model matching the upper part of the transition zone from the deterministic model.

Table 1. Standard deviation average from the posterior distribution of parameters z_1 and z_2 for the MT stations inverted.

$\lambda(\text{m})$	$\bar{\sigma}_{z_1}(\text{m})$	$\bar{\sigma}_{z_2}(\text{m})$
No prior	31	77
80	30	71
60	28	68
40	27	64
20	27	54

Each row presents values for different λ (see the “Methodology” section, equation 2).

DISCUSSION

Advantages of uncertainty quantification

The clearest advantage of the stochastic inversion we have presented is that it does not give a single, definite view of the subsurface. Instead, inferred reality is presented as a bounded distribution of probable clay locations (Figures 6 and 7). This uncertainty is useful for drilling campaigns because it sets expectations when searching for a reservoir beneath the conductor and it provides a rationale to suspend drilling if the expected resource is not encountered.

Furthermore, analysis of the resource size and economic potential during the field exploration stage is more nuanced when qualified with uncertainty. Indeed, the conceptual modeling process for geothermal fields is improved by uncertainty because this admits the possibility of several competing models (consistent with the uncertain limits of the inversion) and maintains open-mindedness within the exploration team. The spatial distribution of uncertainty can also be useful when designing new acquisition campaigns because these can be targeted in locations where new data can optimally fill blind spots.

A second advantage of stochastic inversion is the ability to explore nonuniqueness when trading off layer thickness and resistivity. Selecting a single model from a large pool of possible models could incorrectly identify conductor thickness, compromising the operational decision making — drilling, resource estimation — that follows. In these instances, it is better to define and present the set of possible models (as we have done here) particularly because the use of structural priors based on lithologic information (such as MeB data) can be used to further restrict the uncertainty.

Computational cost of stochastic and deterministic inversions

We have recorded computational runtime for stochastic and deterministic inversion methods for synthetic and field examples. For the synthetic example (Figure 6), the stochastic inversion is slightly faster than the 2D deterministic inversion, although both are completed in a reasonable time on a desktop computer (Table 2). For this example, the computational expense depends on the parameters of both inversions (the cutoff rms misfit for the deterministic method is set at 1.0, and the number of walkers and jumps is set to 36 and 3500 for the stochastic method).

For the field example, our stochastic inversion obtains a more substantial speedup over the 3D deterministic inversion (Table 2) because it focuses resources on a collection of small models with five parameters per station. Each station inversion is independent, and this step is fully parallelized. In contrast, 3D deterministic inversions use a dense grid with thousands of parameters (10^5 for the Wairakei example) and are understandably more resource-intensive. However, 3D inversions are capable of imaging resistivity distributions to great depths and are required in the presence of 2D and 3D structures. Without further refinements, our 1D stochastic inversion may ultimately be limited to imaging shallow 1D features such as the horizontally extensive conductors that we have demonstrated here. Nevertheless, for this task, it is useful to have a tool that reprioritizes computational resources from model complexity to model uncertainty.

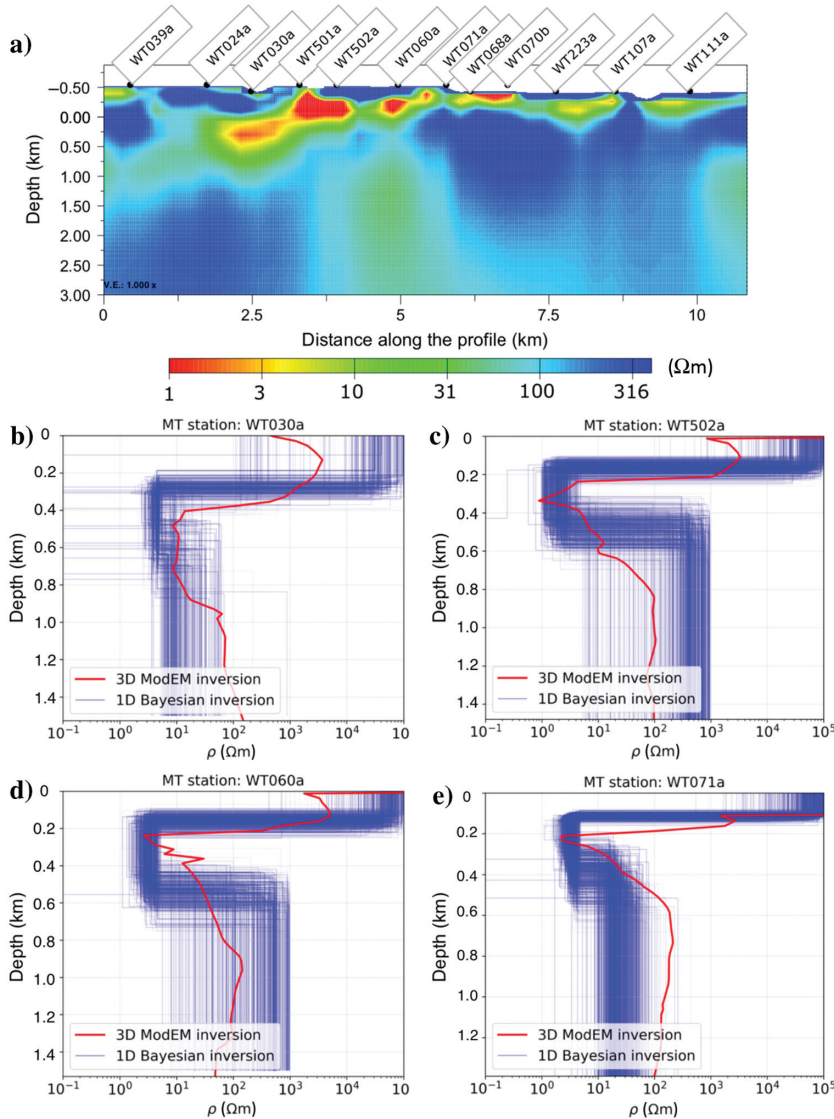


Figure 8. (a) A vertical slice of the optimal resistivity model from a 3D deterministic inversion using ModEM (Kelbert et al., 2014) for the Wairakei MT profile (Figure 1). The location of the 12 inverted MT stations is shown in boxes over the topography. (b-e) Resistivity profiles beneath four stations interpolated from the deterministic 3D resistivity model are compared with the conductor boundary estimated from the MT Bayesian inversion for stations (b) WT030a, (c) WT502a, (d) WT060a, and (e) WT071a. The red lines show the resistivity profile interpolated at each station position from the 3D resistivity model estimated in the deterministic inversion. The blue lines are three-layer resistivity models sampled from the posterior distribution from the MT Bayesian inversion.

Table 2. Elapsed times for different inversion methods and scenarios.

Inversion	Number of stations	Number of cores	Time (h)
Synthetic 2D ModEM	20	4	~1
Synthetic 1D stochastic	20	4	~0.4
Field 3D ModEM	12	56	~28
Field 1D stochastic	12	4	~0.3

The stochastic synthetic inversion used 36 walkers, each completing 3500 jumps for 1.26×10^5 forward model evaluations. The deterministic synthetic inversion set the rms tolerance for convergence to 1. Both inversions were run on the same desktop computer (four cores). The same parameters were used for the field example, except that the 3D deterministic inversion was performed on a cluster with 56 cores. Note that the 3D ModEM inversion is inverted for the full range of periods (not just the ones that satisfy the 1D assumption as in our stochastic inversion) because this is standard in geothermal exploration when mapping the clay cap through MT inversions. Inverting for fewer periods would decrease the inversion times but would remain in the scale of several hours using the cluster of 56 cores.

Methodological assumptions and limitations

The 1D approximation is essential to the stochastic inversion workflow but is only valid when the MT data exhibit 1D behavior. This has been the case for the shallow sections of geothermal reservoirs such as Wairakei, where the data have a clear 1D character over the first 1–2 km (Figure 5). By establishing a range of periods that satisfy the 1D assumption, our estimates are coherent with the model used. If 2D/3D MT data are observed at depths at which the conductor is inferred to lie, our method would introduce an unconstrained bias in the estimation of boundary depths.

Another assumption of our approach is the use of a three-layer resistivity model that averages the properties of the conductor across a single layer. Resistivity variations inside the layer are therefore not captured, which will pose difficulties in the presence of complex layering. However, due to the diffusive nature of MT signals in conductive layers, stochastic and deterministic inversions are likely to struggle to resolve such layer contrasts, providing they are small.

CONCLUSION

We have presented a 1D stochastic inversion of MT data constrained by lithostratigraphic information from geothermal wells. The method adds new depth and nuance to inversions for shallow conductive bodies in geothermal fields because it focuses on inference under uncertainty. Incorporating clay indicators (MeB) as structural priors reduces uncertainty in the inversion and explicitly represents the irreducible uncertainty as estimated depth intervals for the inferred clay cap boundaries.

A synthetic problem shows significant advantages of the method over deterministic inversions, particularly when MeB data exist to constrain the depth for the bottom of the conductor. Although deterministic inversions also capture the main features of the shallow conductor (shape and resistivities), the uncertain envelope derived from the stochastic inversion delivers more useful information for operational activities that involve risk assessment (drilling, resource estimation). Inversion of field data shows how the introduction of a structural prior into the inversion helps to discriminate between

plausible thicknesses for the conductor and reduces the uncertainty when identifying the lower boundary. We have also applied the technique to a data set from the Wairakei geothermal field, inverting a 2D profile that captures uncertain envelopes for the conductor boundary inside and outside the field.

Overall, comparing our method to deterministic inversions from ModEM demonstrates that the technique is a fast, simple, and effective alternative for problems with a dominantly 1D character. In particular, there are substantial computational savings compared to 3D inversions with the advantage that uncertain envelopes are output for key model features. Future directions for this research could include extending the approach to 2D or 3D forward models, alternative methodologies for approximating the posterior, or incorporation of other geologic data sets as priors.

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DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.

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Biographies and photographs of the authors are not available.